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# On scale dependence of QCD string operators *

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**Abstract**

We have obtained a general solution of evolution equations for QCD twist-2 string operators in form of expansion over complete set of orthogonal eigenfunctions of evolution kernels in coordinate-space representation. In the leading logarithmic approximation the eigenfunctions can be determined using constraints imposed by conformal symmetry. Explicit formulae for the LO scale-dependence of quark and gluon twist-2 string operators are given.

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## Introduction

Recently, there has been a renewed interest in QCD evolution of skewed parton distributions [1, 2]. Skewed parton distributions play a crucial role in description of hard, exclusive QCD processes [1, 2, 3, 4, 5] which are actively considered [6, 7, 8, 9] as tools for investigation of new aspects of non-perturbative QCD dynamics [10, 11]. However, it is clear that before a non-perturbative information can be reliably extracted from experimental data, all perturbative aspects, such as QCD evolution, have to be understood. So far, the main effort has been devoted to studies of evolution equations for skewed parton distributions in the momentum representation [1, 2, 5, 12, 13, 14, 15, 16, 17, 18]. This is certainly the most natural choice in the forward, deep inelastic scattering limit, but, as observed recently [19], as soon as there is a longitudinal momentum transfer between initial and final hadron states, the corresponding amplitude can be as conveniently represented in terms of momentum- as coordinate-space skewed parton distributions.

In a recent paper [19] we have considered evolution of charge-conjugation odd and even flavor non-singlet coordinate-space skewed quark distributions in the leading-logarithmic (LO) approximation [20]. The resulting solutions have form of a Neumann series expansions [21] over a set of Bessel functions of the first kind $J_{1/2+j}$, with $j$ assuming odd, respectively even integer values. That the Bessel functions enter the game is not surprising at all – it has been known for a long time [20] that they form a representation of the conformal group in the coordinate-space and, as this symmetry is preserved at the LO [22, 23], they simultaneously diagonalize corresponding evolution kernels. What is more interesting is that the resulting expansions are valid uniformly for all values of the asymmetry parameter, in contrast to the situation in the momentum-space. There, one has to distinguish between cases where momentum fraction is smaller, respectively larger than the asymmetry parameter. A compact formula, based on the expansion in terms of local, multiplicatively renormalizable operators, exists only in the former case [2, 13, 15, 16, 19].

Evolution equations for twist-2 string operators have been known for a long time [20, 24]. Written in the coordinate-space representation, they have been considered so far as an intermediate step in derivation of evolution equations for skewed parton distributions in the momentum space. However, given the transparent form of the solution to the problem of scale-dependence of coordinate-space skewed parton distributions found in [19] it is natural to ask whether similar formulae can be obtained for evolution of twist-2 string operators themselves. As we demonstrate below, this problem has a positive answer. The main goal of the present paper is to discuss an explicit solution to the evolution equations for string operators in a form of expansion over orthogonal eigenfunctions of corresponding evolution kernels.

As we consider expansion of a non-local operator in terms of an orthogonal set of functions, it is natural that coefficients of such an expansion are themselves non-local operators. Their scale dependence is governed by a set of evolution equations which in general result in a mixing between coefficients corresponding to different orthogonal eigenfunctions. However, this term is present only when the analysis goes beyond the

leading order (LO) approximation.

The remaining presentation is organized as follows. First, we introduce a general solution to QCD evolution equations for twist-2 string operators in a form of expansion over a set of orthogonal eigenfunctions of evolution kernels, find expansion coefficients and discuss their scale-dependence. Next, we perform an explicit analysis of the evolution equations in the LO approximation. The crucial point here is that appropriate sets of orthogonal eigenfunctions can be determined using constraints imposed by the conformal symmetry, .i.e. without solving the eigenvalue problems. In this approximation, the expansion coefficients are identical with conformal string operators introduced in [20]. As a result, we obtain explicit expressions for QCD evolution of quark and gluon string operators. Finally, we summarize.

**Evolution equations for QCD string operators**

Let us start from a discussion of a general structure of QCD evolution equations for string operators. As in Ref.[20], we introduce the following definitions of quark, respectively gluon twist-2 operators

$$O(\alpha,\beta) = \bar{q}(\frac{\alpha+\beta}{2}z)\hat{z}\left[\frac{\alpha+\beta}{2}z, \frac{\alpha-\beta}{2}z\right] q(\frac{\alpha-\beta}{2}z)\,, \tag{1}$$

$$G(\alpha,\beta) = z_\alpha G^a_{\mu\alpha}(\frac{\alpha+\beta}{2}z)\left[\frac{\alpha+\beta}{2}z, \frac{\alpha-\beta}{2}z\right]_{ab} G^b_{\mu\beta}(\frac{\alpha-\beta}{2}z)z_\beta\,. \tag{2}$$

In the above formula $z$ is a light-like vector, $z^2 = 0$. The square brackets denote the path-ordered exponential:

$$[az, bz] = \mathcal{P}\exp[-ig\int_a^b z_\mu A^\mu(tz)\,dt] \tag{3}$$

which ensures gauge-invariance of the above definitions. Note that $\alpha\, z$ describes the center of the string composed from quark fields and the gluon line between them while $\beta\, z$ corresponds to its "length", understood simply as the difference between coordinates of quark or gluon fields at the string ends.

Charge conjugation odd and even quark string operators are obtained from $O(\alpha,\beta)$ by taking its components symmetric, respectively antisymmetric in $\beta$. Denoting the former and the latter by $O^+(\alpha,\beta)$, respectively $O^-(\alpha,\beta)$ we define

$$\begin{aligned} O^+(\alpha,\beta) &= \frac{1}{2}\left(O(\alpha,\beta)+O(\alpha,-\beta)\right)\,,\\ O^-(\alpha,\beta) &= \frac{i}{2}\left(O(\alpha,\beta)-O(\alpha,-\beta)\right)\,. \end{aligned} \tag{4}$$

Similarly to local operators, bare string operators have UV-divergences and should be renormalized. In complete analogy with the former case one can define renormalized string operator through the relation:

$$O_R(\omega) = Z(\omega,\omega')\otimes O_B(\omega')\,. \tag{5}$$

Here we have introduced a compact notation $\otimes$ for the two-dimensional integral

$$\int_{-\infty}^{\infty} d\alpha \int_{0}^{\infty} d\beta$$

and $\omega \equiv \{\alpha, \beta\}$. Function $Z(\omega, \omega')$ corresponds to a renormalization constant in a local case. We suppose that in MS-scheme it has the following structure:

$$Z(\omega, \omega') = \delta(\omega - \omega') + \sum_{n \geq 1} \frac{1}{\varepsilon^n} \sum_{k \geq n} a_s^k Z_{nk}(\omega, \omega') \,, \tag{6}$$

where $a_s = \frac{\alpha_s(\mu^2)}{4\pi}$ and $\varepsilon = 2 - d/2$, $d$ being the space-time dimension. Coefficients $Z_{nk}$ can be calculated in perturbation theory. In the MS-like schemes they do not depend on the renormalization scale $\mu$.

The renormalization group equation can be derived by taking a total derivative over $\mu$ of the left- and right-hand sides of (5). In this way one arrives at an evolution equation for the renormalized string operator $O_R(\omega)$ [20, 24]

$$\mu \frac{d}{d\mu} O_R(\omega) = a_s V(\omega, \omega') \otimes O_R(\omega') \,. \tag{7}$$

The evolution kernel $V(\omega, \omega')$ is defined as

$$\begin{aligned} a_s V(\omega, \omega') &= \mu \frac{\mu}{d\mu} Z(\omega, \omega'') \otimes Z^{-1}(\omega'', \omega') \,. \\ Z^{-1}(\omega'', \omega') \otimes Z(\omega', \omega) &= \delta(\omega'' - \omega) \,. \end{aligned} \tag{8}$$

Although it has not been indicated explicitly, $V(\omega, \omega')$ is a function of the QCD coupling constant:

$$V(\omega, \omega') = \sum_{n \geq 0} a_s^n(\mu^2) V^{(n)}(\omega, \omega') \,, \tag{9}$$

except of the leading order (LO) approximation, when it is given simply by a function $V^{(0)}(\omega, \omega')$.

After these preliminaries, we shall construct now a general solution of the operator evolution equation (7). To this end, let us assume that one can find a solution of the following system of equations for eigenfunctions and corresponding eigenvalues of the evolution kernel:

$$\begin{aligned} V(\omega, \omega') \otimes \varphi_i(\omega') &= \gamma_i \varphi_i(\omega') \\ \bar{\varphi}_i(\omega) \otimes V(\omega, \omega') &= \gamma_i \bar{\varphi}_i(\omega) \end{aligned} \tag{10}$$

In the theory of integral equations the second equation is called conjugated to the first one [25].

Eigenfunctions $\varphi_i(\omega)$ and $\bar{\varphi}_j(\omega)$ are orthogonal with respect to the scalar product $\otimes$, i.e.

$$\langle \bar{\varphi}_i, \ \varphi_j \rangle = \bar{\varphi}_i(\omega) \otimes \varphi_j = \delta_{ij} N_i \ , \tag{11}$$

where $N_i$ is some normalization constant. Note that beyond the leading order eigenfunctions $\varphi_i(\omega)$, $\bar{\varphi}_j(\omega)$ and eigenvalues $\gamma_i$ depend on the running QCD coupling because the kernel $V(\omega,\omega')$ itself becomes a function of $a_s(\mu^2)$, see eq.(9).

Solution of eq.(7) can be written in the following form:

$$O_R(\omega) = \sum_{n\geq 0} S_n \; \varphi_n(\omega)\,. \tag{12}$$

The operator-valued coefficients $S_n$ can be obtained by taking the scalar products of both sides of the above equation with eigenfunctions $\bar{\varphi}_n$:

$$S_n = \frac{1}{N_n}\,\bar{\varphi}_n(\omega')\otimes O_R(\omega') \tag{13}$$

As it follows, $S_n$ themselves are non-local operators. Moreover. eq.(7) leads to the following set of evolution equations for operators $S_n$:

$$\mu\frac{d}{d\mu}S_n + \sum_{m\geq 0} S_m\langle\bar{\varphi}_n,\;\dot{\varphi}_m\rangle = a_s\,\gamma_n S_n,\;\;\dot{\varphi}_n \equiv \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\varphi_n\,. \tag{14}$$

The second term on the left-hand side induces mixing between operators corresponding to different orthogonal eigenfunctions. It is absent in the LO because eigenfunctions $\varphi_n$ do not depend on $\alpha_s$ in this approximation. As a consequence operators $S_n$ become multiplicatively renormalizable. This case will be discussed in more details in the next section.

**LO evolution of string operators**

In the following we will follow notation of [20]. To the leading-order accuracy, the evolution kernel is given by $V(\omega,\omega') = V^{(0)}(\omega,\omega')$ and is therefore independent on $a_s(\mu)$. Arguments based on the conformal symmetry can be used to find explicit solutions of the eigenvalue equation (10). In Ref.[20] the nonlocal conformal operators were introduced for the first time and the conformal symmetry arguments were used to find the solution of (7) in terms of an integral representation over a complex conformal spin $j$. Our goal here is to represent a solution in the form of an expansion (12). Is is equivalent to the former one, but much simpler from the numerical point of view.

Now, let us consider solutions to the LO evolution equations in the framework set up in the previous section. First, note that the two-dimensional eigenvalue problem (10) can be reduced to a one-dimensional one by using translational invariance to separate the dependence on the variable $\alpha$, which describes the position of the center-of-mass of the string. This is done by substituting

$$\phi_j(\alpha,\beta) = e^{ik\alpha}f_j(k\beta),\;\;\bar{\phi}_j(\alpha,\beta) = e^{ik\alpha}\bar{f}_j(k\beta)\,. \tag{15}$$

In this way, $\phi_j(\alpha,\beta)$ and $\bar{\phi}_j(\alpha,\beta)$ can be interpreted as wave functions of bound systems of two particles, with a momentum $k$ and an internal motion described by $f_j(k\beta)$ and $\bar{f}_j(k\beta)$, respectively. Let us first consider the case of flavor non-singlet quark operators.

Inserting (15) into the eigenvalue equations (10) one arrives at the following system of equations [20]:

$$
\begin{aligned}
H_1 f_j &\equiv \int_0^{\rho_2} V(\rho_1,\rho_2) f_j(\rho_1) = \gamma_j f_j(\rho_2), \\
H_2 \bar{f}_j &\equiv \int_{\rho_1}^{\infty} V(\rho_1,\rho_2) \bar{f}_j(\rho_2) = \gamma_j \bar{f}_j(\rho_1)\,.
\end{aligned}
\tag{16}
$$

where $\rho \equiv k\beta$. The reduced kernel $V(\rho_1,\rho_2)$ is given by

$$
V(\rho_1,\rho_2) = -3\delta(\rho_1-\rho_2) + \frac{2\sin(\rho_1-\rho_2)}{\rho_2} + \frac{4\rho_1\cos(\rho_1-\rho_2)}{\rho_2(\rho_1-\rho_2)} + 4\delta(\rho_1-\rho_2)\int_0^{\rho_2}\frac{d\rho}{\rho}\,. \tag{17}
$$

Equations (16) have the same form as the ERBL-equation for evolution of the pion distribution amplitude [22, 26]. The latter is defined as the reduced matrix element of twist-2 non-local quark operator between the pion state and the vacuum. Conformal symmetry was shown to play a crucial role in finding solution of the LO ERBL equation a long-time ago [22, 26, 23]. In complete analogy, we shall now demonstrate how the symmetry considerations allow to determine eigenfunctions $f_j(k\beta)$ and $\bar{f}_j(k\beta)$. Note that in the present case the evolution is considered at the operator level, without any reference to matrix elements.

$\mathbf{J}^2$, the Casimir operator of the collinear conformal group SO(2,1), can be represented as an differential operator acting on the parameters $\alpha, \beta$. After substitution (15) one obtains [20]:

$$
\begin{aligned}
\mathbf{J}^2 f_j(\rho) &\equiv L_1 f_j(\rho) = \left[\rho^2\frac{d^2}{d\rho^2} + 4\rho\frac{d}{d\rho} + \rho^2k^2 + 2\right] f_j(\rho) \\
\mathbf{J}^2 \bar{f}_j(\rho) &\equiv L_2 \bar{f}_j(\rho) = \left[\rho^2\frac{d^2}{d\rho^2} + \rho^2k^2\right] \bar{f}_j(\rho)\,.
\end{aligned}
\tag{18}
$$

Note that the explicit form of $J^2$ depends on whether it acts on $f_i(k\beta)$ or on $\bar{f}_i(k\beta)$. Using equations (16) and (18) it is easy to check by an explicit calculation that $\mathbf{J}^2$ indeed commutes with the 'Hamiltonians' $H_{1,2}$, as required by the symmetry arguments. As it follows, the eigenfunctions of $H_{1,2}$ have to be the same as the eigenfunctions of $L_{1,2}$:

$$
L_1 f_j(\rho) = j(j+1) f_j(\rho),\;\; L_2\bar{f}_j(\rho) = j(j+1)\bar{f}_j(\rho)\,, \tag{19}
$$

where $j$ are positive integers. As $L_1$ and $L_2$ are related by

$$
L_2\rho^2 = \rho^2 L_1 \tag{20}
$$

one obtains the following relation between $f_j(\rho)$ and $\bar{f}_j(\rho)$

$$
\bar{f}_j(\rho) = \rho^2 f_j(\rho)\,. \tag{21}
$$

Solutions of the set of differential equations (19) are given by Bessel functions:

$$
f_j(\rho) = c_j(k)\rho^{-3/2} Z_{j+1/2}(\rho), \bar{f}_j(\rho) = \bar{c}_j(k)\rho^{1/2} Z_{j+1/2}(\rho)\,, \tag{22}
$$

where $c_j(k)$, $\bar{c}_j$ are yet undetermined normalization constants. As the operator $O_R(\omega)$ has a well-defined local limit as $\beta \to 0$, the validity of expansion (12) requires that $f_j(\beta)$ are regular in this limit as well. In terms of $f_j$ and $\bar{f}_j$ the scalar product (11) reduces to

$$\langle \bar{f}_j(\rho) f_i(\rho) \rangle = \int_0^\infty d\rho \bar{f}_j(\rho) f_j(\rho) = N_i \delta_{ij}\,, \tag{23}$$

and therefore $f_j$ and $\bar{f}_j$ must be sufficiently regular such that the integral exists. These conditions are satisfied when we identify $Z_{j+1/2}(\rho)$ with $J_{j+1/2}(\rho)$, the Bessel functions of the first kind. In particular, the scalar product (23) reduces to the well known integral ($\{i,\ j\} = 1, 2, 3, ...$) [27]:

$$\langle \bar{f}_i(\rho) f_j(\rho) \rangle = \bar{c}(k)_i c_j(k) \int_0^\infty \frac{d\rho}{\rho} J_{j+1/2}(\rho) J_{i+1/2}(\rho) = \bar{c}(k)_i c_j(k) \frac{\delta_{ij}}{2i+1}\,. \tag{24}$$

For reasons which will shortly become clear we set $\bar{c}_j(k) = k^{-1/2}$ and $c_j(k) = k^{3/2}$. With this choice the nonlocal operators $S_j$ defined according to (13) assume the form:

$$S_j \equiv S(1/2+j, k; \mu^2) = \int_{-\infty}^\infty d\alpha\, e^{ik\alpha} \int_0^\infty d\beta\, \sqrt{\beta} J_{1/2+j}(|k|\beta)\, O(\alpha,\beta)_{\mu^2}\,. \tag{25}$$

As noted above, they are multiplicatively renormalizable [20]:

$$\begin{aligned}
&\mu \frac{d}{d\mu} S(1/2+j, k; \mu^2) = a_s \gamma_j S(1/2+j, k; \mu^2),\ S(1/2+j, k; \mu^2) = L_j S(1/2+j, k; \mu_0^2),\\
&L_j \equiv \left(\tfrac{\log(\mu^2/\Lambda^2)}{\log(\mu_0^2/\Lambda^2)}\right)^{-\frac{\gamma(j)}{b_0}},\ \gamma(j) \equiv \gamma_{QQ} = C_F\,\left(3 + \tfrac{2}{j(j+1)} - 4(\Psi(j+1) + \gamma_{\rm E})\right)\,.
\end{aligned} \tag{26}$$

Here, $\gamma(j)$ is the corresponding anomalous dimension known, e.g., from analysis of deep-inelastic scattering process.

Now we are in position to write down the decomposition (12) explicitly:

$$O(\alpha,\beta)_{\mu^2} = \int_{-\infty}^\infty \frac{dk}{2\pi}\, e^{-ik\alpha} \beta^{-\frac{3}{2}} \sum_{j=1}^\infty (1+2j) L_j J_{1/2+j}(|k|\beta) S(1/2+j, k; \mu_0^2) \tag{27}$$

It remains only to check the normalization. As $\mu^2 = \mu_0^2$ and therefore $L_j = 1$ for all $j$ we should have

$$O(\alpha,\beta) = \int_{-\infty}^\infty d\alpha' \int_0^\infty d\beta' \mathcal{R}(\alpha,\beta|\alpha',\beta')\, O(\alpha',\beta') \tag{28}$$

with

$$\mathcal{R}(\alpha,\beta|\alpha',\beta') = \int \frac{dk}{2\pi} e^{ik(\alpha'-\alpha)} \beta^{-\frac{3}{2}} \beta'^{\frac{1}{2}} \sum_{j=1}^\infty (1+2j) J_{1/2+j}(|k|\beta) J_{1/2+j}(|k|\beta') = \delta(\alpha-\alpha')\delta(\beta-\beta')\,. \tag{29}$$

To this end, recall that the series

$$\beta^{-\frac{3}{2}} \beta'^{\frac{1}{2}} \sum_{j=1}^\infty (1+2j) J_{1/2+j}(|k|\beta) J_{1/2+j}(|k|\beta') = \delta(\beta-\beta') \tag{30}$$

does not depend on $k$ and represents the Neumann expansion of a $\delta$-function in terms of Bessel functions [19, 21]. Equation (29) follows immediately. Note that this observation explains also the choice of the coefficients $c_i$, $\bar{c}_i$.

Equation (27) describes solution of the LO evolution equation as an expansion in terms of orthogonal eigenfunctions of the evolution kernel. On the other hand, it can be interpreted as an expansion of a quark string operator in terms of conformal string operators. Series expansion in terms of Bessel functions is known in mathematical literature as the Neumann series, see [21] for more detailed information. Evolution of flavor non-singlet, charge-conjugation odd and even quark string operators can be obtained from (27) by taking its symmetric, respectively antisymmetric in $\beta$ components.

Conformal operators (25) are defined only for positive, integer values of the conformal spin $j$. On the other hand, the solution found in [20] is based on an integral representation which requires analytical continuation of $j$ into the complex plane. In our previous paper [19] we have explicitly checked that integration over $j$ indeed reproduces the series (27).

Let us now turn our attention to operators with flavor singlet and positive charge parity quantum numbers. Here we have to consider mixing between gluon and quark string operators, see equations (2) and (4) for corresponding definitions. Note that the quark operator $Q \equiv O^-$ receives contribution from all $N_F$ active flavors. An analysis analogous to the flavor non-singlet case leads to the following basis of the nonlocal conformal quark and gluon operators:

$$
\begin{aligned}
S_G(1/2+j,k;\mu^2) &= \int_{-\infty}^{\infty} d\alpha\, e^{ik\alpha} \int_0^{\infty} d\beta\, \beta^{3/2} J_{1/2+j}(|k|\beta)\, G(\alpha,\beta)_{\mu^2}\,, \\
S_Q(1/2+j,k;\mu^2) &= \int_{-\infty}^{\infty} d\alpha\, e^{ik\alpha} \int_0^{\infty} d\beta\, \sqrt{\beta} J_{1/2+j}(|k|\beta)\, Q(\alpha,\beta)_{\mu^2}\,.
\end{aligned}
\tag{31}
$$

Here $j = 2+2n$, $n = 0,1,2,3,...$ is the conformal spin. Operators $S_G(1/2+j,k;\mu^2)$ and $S_Q(1/2+j,k;\mu^2)$ depend on the renormalization scale according to

$$
\begin{aligned}
S_G(1/2+j,k;\mu^2) = {}& [\lambda_+ L_+ - \lambda_- L_-]\, S_G(1/2+j,k;\mu_0^2) + \\
& + \gamma_{GQ} \frac{(j-1)}{\sqrt{D}} [L_+ - L_-]\, S_Q(1/2+j,k;\mu_0^2)\,,
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
S_Q(1/2+j,k;\mu^2) = {}& [\lambda_+ L_- - \lambda_+ L_-]\, S_Q(1/2+j,k;\mu_0^2) + \\
& + \frac{\gamma_{QG}}{(j-1)} \frac{1}{\sqrt{D}} [L_+ - L_-]\, S_G(1/2+j,k;\mu_0^2)\,.
\end{aligned}
\tag{33}
$$

Here we have introduced a shorthand notation:

$$
\begin{aligned}
& L_\pm = \left( \frac{\log(\mu^2/\Lambda^2)}{\log(\mu_0^2/\Lambda^2)} \right)^{-\frac{\gamma_\pm(j)}{b_0}}, \; \lambda_\pm = \frac{1}{2}\frac{1}{\sqrt{D}}[\gamma_{GG} - \gamma_{QQ} \pm \sqrt{D}], \\
& \gamma_\pm = \frac{1}{2}\left( \gamma_{GG} + \gamma_{QQ} \pm \sqrt{D} \right), \; D = (\gamma_{GG} - \gamma_{QQ})^2 + 4\gamma_{GQ}\gamma_{QG}\,.
\end{aligned}
\tag{34}
$$

In convention adopted in this paper the anomalous dimensions $\gamma_{GG}, \gamma_{GQ}, \gamma_{QG}$ differ by a sign from those given in Ref.[20]. Note that although not explicitly indicated, $j$-dependence of the anomalous dimensions is implied here.

To find the evolution of the string operators $G(\alpha,\beta)$, $Q(\alpha,\beta)$ we have to rewrite them as series expansions in terms of the conformal string operators (31). Consider first the gluon operator $G(\alpha,\beta)$. Using the Neumann expansion of the $\delta$-function:

$$\beta\delta(\beta-\beta') = \sum_{n=0}^{\infty}(2\nu+2+4n)J_{\nu+1+2n}(|k|\beta)J_{\nu+1+2n}(|k|\beta')\,, \tag{35}$$

one easily obtains the desired formula:

$$\begin{aligned} G(\alpha,\beta;\mu^2) &= \int_{-\infty}^{\infty} d\alpha' \int_0^{\infty} d\beta'\,\delta(\alpha-\alpha')\delta(\beta-\beta')G(\alpha',\beta';\mu^2) = \\ &= \int d\alpha' \int d\beta' \int \frac{dk}{2\pi} e^{ik(\alpha'-\alpha)}\beta^{-\frac{5}{2}} \sum_{n=0}^{\infty}(5+4n)J_{\frac{5}{2}+2n}(|k|\beta)J_{\frac{5}{2}+2n}(|k|\beta')\beta'^{\frac{3}{2}}G(\alpha',\beta';\mu^2) = \\ &= \int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{-ik\alpha}\beta^{-5/2}\sum_{n=0}^{\infty}(5+4n)J_{\frac{5}{2}+2n}(|k|\beta)S_G(\tfrac{5}{2}+2n,k;\mu^2)\,. \end{aligned} \tag{36}$$

Taking into account (32) one finally arrives at the following expression for the evolution of the gluon operator:

$$\begin{aligned} G(\alpha,\beta;\mu^2) = \int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{-ik\alpha}\beta^{-5/2}\sum_{n=0}^{\infty}(5+4n)J_{\frac{5}{2}+2n}(|k|\beta)\times \\ \times\Big\{\left[\lambda_+L_+-\lambda_-L_-\right]_{(j=2n+2)}S_G(\tfrac{5}{2}+2n,k;\mu_0^2)+ \\ +\gamma_{GQ}\frac{2n+1}{\sqrt{D}}\left[L_+-L_-\right]_{(j=2n+2)}S_Q(\tfrac{5}{2}+2n,k;\mu_0^2)\Big\}\,. \end{aligned} \tag{37}$$

Evolution of the quark-singlet operator can be found in a similar way. First, we establish an expansion in terms of non-local conformal operators

$$Q(\alpha,\beta;\mu^2) = \int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{-ik\alpha}\beta^{-3/2}\sum_{n=0}^{\infty}(5+4n)J_{\frac{5}{2}+2n}(|k|\beta)S_Q(\tfrac{5}{2}+2n,k;\mu^2)\,. \tag{38}$$

The resulting solution to the evolution equations reads

$$\begin{aligned} Q(\alpha,\beta;\mu^2) = \int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{-ik\alpha}\beta^{-3/2}\sum_{n=0}^{\infty}(5+4n)J_{\frac{5}{2}+2n}(|k|\beta)\times \\ \times\Big\{\left[\lambda_+L_--\lambda_-L_+\right]_{(j=2n+2)}S_Q(\tfrac{5}{2}+2n,k;\mu_0^2)+ \\ +\frac{\gamma_{QG}}{(2n+1)}\frac{1}{\sqrt{D}}\left[L_+-L_-\right]_{(j=2n+2)}S_G(\tfrac{5}{2}+2n,k;\mu_0^2)\Big\}\,. \end{aligned} \tag{39}$$

Equations (27), (37) and (39) are new and represent the main results of this paper. As advocated in the previous section, they describe the scale dependence of QCD string operators in form of expansion over a set of eigenfunctions of corresponding evolution kernels. Evaluating matrix elements between appropriate nucleon states one finds the explicit form of scale dependence of skewed parton distributions in the coordinate-space representation [19]. Due to translational invariance, the integral over $k$ can be trivially performed and one is left with an expansion of corresponding coordinate-space skewed parton distributions in terms of orthogonal set of Bessel functions, valid for all values of the asymmetry parameter. Hence, the solution in the coordinate-space is much more transparent than in the momentum representation.

**Summary**

We have argued that solutions of evolution equations for twist-2 QCD string operators can be written as series expansions in terms of the orthogonal sets of eigenfunctions of evolution kernels. We have obtained a general form of evolution equations for operator-valued expansion coefficients and found that in a general case evolution results in a mixing between coefficients corresponding to different eigenfunctions.

In the LO approximation the eigenfunctions of evolution kernels can be found explicitly using conformal symmetry arguments and, at the same time, evolution equations for expansion coefficients become much simpler than in the most general case. As a result the expansion coefficients can be identified with conformal string operators. Those with flavor non-singlet or charge-parity odd quantum numbers are simply multiplicatively renormalized and those corresponding to flavor singlet and positive charge-parity are subject only to the usual mixing between gluon and quark operators. In both cases explicit formulae for scale dependence of quark and gluon string operators have been derived.

Taking corresponding matrix elements one obtains immediately solution to the problem of scale dependence of skewed parton distributions in the coordinate-space. Here, contrary to the situation in momentum space, the resulting expansion over a set of orthogonal eigenfunction is valid for all values of the asymmetry parameter.

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